Written Exam for the B.Sc. in Economics, Winter 2010/2011

Microeconomics C

Final Exam

January 4, 2011

(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

PLEASE ANSWER ALL QUESTIONS BELOW.
PLEASE EXPLAIN YOUR ANSWERS.

1. (a) Find *all* (pure and mixed) Nash equilibria of the following game:

	L	R
T	10,5	11,4
B	9,6	12,6

(b) Consider the following two-stage game with two players. If the second stage is reached, the two players play the following static game (player 1 chooses a row, player 2 chooses a column):

	C	D
A	0,0	3,1
B	1,3	2,2

In the first stage, player 1 decides whether to proceed to the second stage (action Y for yes) or not (action N for no). If he plays N then the game ends and each player gets a payoff of 2.

i. Draw a game tree representing the two-stage game. Is it a game of perfect or imperfect information?

ii. How many subgames are there in the game (excluding the game itself)? What are the possible strategies for the two players?

iii. Find all pure strategy subgame perfect Nash equilibria.

iv. Write down the normal-form ("matrix-form") of the game and find all pure strategy Nash equilibria. Are all of them subgame perfect? If not, explain why.

2. Two economists are working on a joint project. Economist i, $i = 1, 2$, exert an effort $y_i \geq 0$. The resulting quality of the project is

$$q(y_1, y_2) = y_1 y_2.$$

Exerting effort is costly. More precisely, the cost functions for the two economists are

$$C_1(y_1) = \frac{1}{3}(y_1)^3$$

and

$$C_2(y_2) = (y_2)^2.$$

The payoff for economist i, U_i, is equal to the quality of the project minus his cost of effort. I.e., we have

$$U_1(y_1, y_2) = q(y_1, y_2) - C_1(y_1)$$

and

$$U_2(y_1, y_2) = q(y_1, y_2) - C_2(y_2).$$

(a) Consider the game where the economists choose their effort levels simultaneously and independently. Derive the best response functions. Find the (pure strategy) Nash equilibrium (y_1^{NE}, y_2^{NE}) with $y_1^{NE}, y_2^{NE} > 0$.

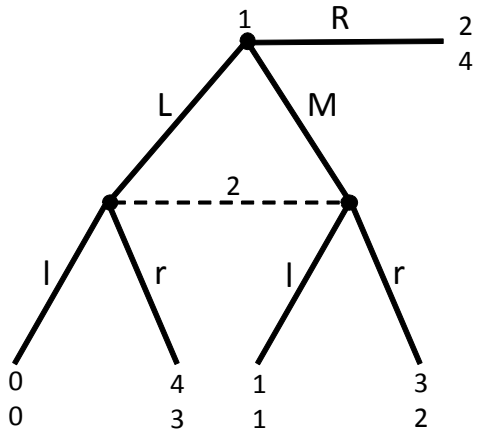
(b) Suppose now that economist 1 chooses his effort first. Economist 2 observes this and then makes his choice of effort. Find the (pure strategy) subgame perfect Nash equilibrium of this game.

(c) Compare the outcomes in (a) and (b) with respect to the payoffs of the economists. Which game does each of the two economists prefer? Give an intuitive explanation of your answer.

(d) Find the socially optimal levels of effort (y_1^{SO}, y_2^{SO}), i.e., the levels that maximize the sum of the two economists' payoffs. Calculate the payoff that each economist gets in the social optimum.

(e) Suppose the game studied in (a) is repeated over an infinite time horizon $t = 1, 2, ..., \infty$. The discount factor is $\delta \in (0, 1)$ and each economist maximizes the sum of discounted payoffs. In this infinitely repeated game, does there exist a subgame perfect Nash equilibrium in which the outcome of each stage is (y_1^{SO}, y_2^{SO})? An intuitive explanation is sufficient.

3. First, consider the following game of imperfect information:



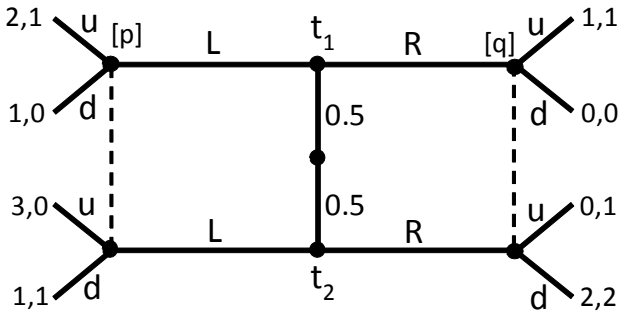
Consider only pure strategies.

(a) Find all Nash equilibria (NE) and all subgame perfect Nash equilibria (SPNE).

(b) Find all perfect Bayesian equilibria (PBE).

(c) Comment on the difference between the set of SPNE and the set of PBE. Which of these two equilibrium concepts do you find most appropriate for this game? Why?

Finally, consider the following signalling game:



(d) Find a separating PBE.